# Random angular coding for superresolved imaging

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In this paper, we present a new approach capable of working under coherent and incoherent illumination for achieving superresolution by random coding of the object's angular information. By placing two static random masks in optically conjugate planes inside an aperture-limited imaging setup, one may obtain a transmitted image containing spatial resolution higher than the one obtained without the masks. As the most noticeable fact, the superresolution effect is obtained without imposing any restrictions either in the time domain or in the field-of-view domain but rather only in the dynamic range of the camera device. Experimental verifications for the proposed technique with incoherent illumination with a low numerical aperture (NA) lens are presented. © 2010 Optical Society of America

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### 1. Introduction

Superresolution is an interesting as well as practical field in which one exceeds the diffraction as well as the geometric limits of the imaging system by imaging smaller spatial features that are not resolvable under the conventional imaging mode by the same imaging system [1-3]. The improvement in resolution is obtained by imposing restrictions in various other domains, such as time [4-6], field of view [7-9], code [10–12], polarization [13–15], wavelength [16– 18], and dynamic range [19]. The operation principle is basically the same for all the strategies. This includes encoding the object's small spatial features (which cannot pass through the imaging system) in the domain in which the restriction is to be imposed and, once they are transmitted through the imager, properly decoding them into the space domain to finally achieve a superresolved image.

Time domain is one of the most appealing approaches to achieve superresolution [4-6,20-25]. In such a case, the spatial-frequency content of the object's spectrum is encoded into the temporal bandwidth degree of freedom because the object's amplitude distribution is temporally restricted, that is, time independent or at least static during the time when performing the approach. Then, different frequency band passes are transmitted through the limited system aperture in different time slots as a consequence of the time multiplexing performed over the object's spectrum. After that, a superresolved image of the object is recovered by means of a proper decoding process. Classically, the encoding/decoding is performed using diffraction gratings (physical or projected) in movement (transversal or axial). However, time multiplexing superresolution needs that the object remains constant during the observation time. This fact disables this approach in the characterization of fast dynamic processes.

On the other hand, superresolution by multiplexing in the object field-of-view domain instead of time domain [7–9,26–29] is also a common approach. Its underlying principle is based on angular coding of the object's small spatial features by adding a restriction over the size of the field-of-view degree of freedom. In other words, additional spatial-frequency content is multiplexed in different positions of the object field of view by using static rather than dynamic encoding/decoding gratings. To allow that fact, the object must be restricted to a limited area in the imaging system field of view to avoid overlapping

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of the different band pass images. Thus, the use of static gratings allows working with temporally moving objects.

In that sense, a very interesting approach avoiding object field-of-view restriction to achieve superresolution was reported in [18]. In that work, the authors proposed the use of two static gratings and polychromatic illumination. The incoherent illumination averages the ghost images obtained outside the region of interest because the positions of those images are wavelength dependent. Thus, no limitation over the object field of view is required any more due to the wavelength averaging. However, the proposed system affects the dynamic range of the detector device because the background intensity level is improved as a consequence of the averaging.

Another approach for superresolved imaging without restricting the field-of-view domain was reported by Zalevsky et al. [30]. The key point is to use the various dilations of the gratings instead of shifting them: instead of moving the grating to allow time shift of the object's spatial-frequency information, they achieve the required encoding/decoding process by spectral dilation, that is, by considering two identical amplitude masks with varying periods along the x- and y-orthogonal directions. In the encoding, the variable period mask provides different scaled replicas of the object's spectrum at the system Fourier plane, allowing the transmission of different spatial-frequency bands in different angles. However, the different spatial-frequency bands appear in different areas of the system aperture due to the local variation of the mask's period: since each local region comes from a different scaled period, the scaling provides orthogonality between the different transmitted band passes. Thus, the role of the second grating is to reposition each transmitted band pass to its proper location in the object's spectrum, that is, to its proper angular direction. Finally, the average along the y direction eliminates the unwanted spectral information while enhances the desired one due to the orthogonal coding/decoding process. Thus, it is possible to retrieve a superresolved image along the x direction. The method reported in [30] is valid for one-dimensional (1D) objects as it does not sacrifice either time or field of view; however, the dynamic range of the output image is lowered due to the averaging along the y axis.

In this paper, we generalize the concept of superresolution based on two static gratings by using two random static masks for the encoding/decoding. Although the use of random patterns to achieve superresolution has been reported previously in the literature [31-34], multiplexing in other domains is also needed. Now, the restriction is only imposed in the dynamic range because the contrast of the obtained superresolved image is reduced. As in the case of static grating approaches, the random masks must have smaller features than those aimed to be resolved in the object because we are implementing angular multiplexing. However, now, we are expanding the concept reported in [30] to the two-dimensional (2D) case while reporting the application of the method not only for coherent but also for incoherent (extended white-light source) illumination. Moreover, the gain in resolution depends on the encoding mask pixel size and a factor of noise but it is independent from the NA of the imaging system. This is an important improvement in comparison to the original superresolving idea considering two fixed variable masks. The achieved experimental results suggest that the technique can be implemented in microscopy by properly selecting the pixel size of the encoding masks to the NA of the objective lens.

The manuscript is organized as follows. In Section 2, we present the mathematical analysis. In Section 3, we perform the numerical simulation results for the suggested system. In Section 4, we perform the experimental validation. Finally, the paper is concluded in Section 5.

# 2. Mathematical Derivation

The schematic sketch of the proposed setup, which is a 4F imaging processor (unity magnification), is shown in Fig. 1. We now analyze it mathematically and prove that indeed a superresolved imaging is obtained. In addition, we show the required trade-offs to obtain the desired outcome. For simplicity, we perform 1D analysis. The extension to 2D is straightforward. Thus, the field distribution after free-space propagation of  $z_1$  is equal to

$$g_z(x) = \int G(\mu) \exp(\pi i \lambda z_1 \mu^2) \exp(2\pi i x \mu) d\mu, \qquad (1)$$

where  $\lambda$  is the wavelength and

$$G(\mu) = \int g(x) \exp(-2\pi i x \mu) dx.$$
 (2)

We multiply this distribution by the random encoding mask denoted by m(x) and the obtained product equals

$$\int \left[ \int M(\mu - \mu_1) G(\mu_1) \right] \\ \times \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \exp(2\pi i x \mu) d\mu, \qquad (3)$$



Fig. 1. (Color online) Theoretical layout of the proposed setup.

while

$$M(\mu) = \int m(x) \exp(-2\pi i x \mu) dx.$$
 (4)

Now we perform back-free-space propagation of  $-z_1$  to obtain the field distribution in the input plane while the effect of the encoding mask is included:

$$\int \left[ \int M(\mu - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right] \\ \times \exp(-\pi i \lambda z_1 \mu^2) \exp(2\pi i x \mu) d\mu.$$
(5)

We progress now to the aperture plane by performing a Fourier transform:

$$\int \left[ \int \left( \int M(\mu - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right) \\ \times \exp(-\pi i \lambda z_1 \mu^2) \exp(2\pi i x \mu) d\mu \right] \\ \times \exp\left(-\frac{2\pi i \mu_2 x}{\lambda F}\right) dx, \tag{6}$$

where F is the focal length of the lens (see Fig. 1). After mathematical simplification of Eq. (6), one obtains

$$\left[\int M\left(\frac{\mu_2}{\lambda F} - \mu_1\right) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1\right] \\ \times \exp\left(-\pi i \lambda z_1 \left(\frac{\mu_2}{\lambda F}\right)^2\right).$$
(7)

We multiply by the aperture (we assume a rect function for the aperture):

$$\left[\int M\left(\frac{\mu_2}{\lambda F} - \mu_1\right) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1\right] \\ \times \exp\left(-\pi i \lambda z_1 \left(\frac{\mu_2}{\lambda F}\right)^2\right) \operatorname{rect}\left(\frac{\mu_2}{\Delta \mu_2}\right), \tag{8}$$

where  $\Delta \mu_2$  is the width of the rectangular aperture. After an additional optical Fourier transform,

$$\int \left[ \int M\left(\frac{\mu_2}{\lambda F} - \mu_1\right) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right] \\ \times \exp\left(-\pi i \lambda z_1 \left(\frac{\mu_2}{\lambda F}\right)^2\right) \operatorname{rect}\left(\frac{\mu_2}{\Delta \mu_2}\right) \\ \times \exp\left(\frac{-2\pi i \mu_2 x}{\lambda F}\right) d\mu_2.$$
(9)

Then, we change the variables into  $\nu = (\mu_2/\lambda F)$ :

$$\int \left[ \int M(\nu - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right] \\ \times \exp(-\pi i \lambda z_1 \nu^2) \operatorname{rect} \left( \frac{\nu}{\Delta \mu_2 / \lambda F} \right) \\ \times \exp(-2\pi i \nu x) d\nu.$$
(10)

Now, we need to have a free-space propagation of  $z_2$  to reach the random decoding mask. To do that we use the angular spectrum approach for computing the free-space propagation, i.e., we multiply the spectrum by the chirp phase factor:

$$\int \left[ \int M(\nu - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right] \\ \times \exp(\pi i \lambda (z_2 - z_1) \nu^2) \operatorname{rect}\left(\frac{\nu}{\Delta \mu_2 / \lambda F}\right) \\ \times \exp(-2\pi i \nu x) d\nu.$$
(11)

After propagating a free-space distance of  $z_2$ , we multiply by the decoding random mask  $m^*(x)$ ,

$$egin{aligned} m^*(x) &= \left(\int M(\mu) \exp(2\pi i x \mu) \mathrm{d} \mu
ight)^* \ &= \int M^*(-\mu) \exp(2\pi i x \mu) \mathrm{d} \mu, \end{aligned}$$

and the expression that we obtain is

$$\begin{split} &\int \left[ \int M(\nu - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right] \\ &\times \exp(\pi i \lambda (z_2 - z_1) \nu^2) \operatorname{rect} \left( \frac{\nu}{\Delta \mu_2 / \lambda F} \right) \\ &\times \left[ \int M^*(-\mu_2) \exp(2\pi i x \mu_2) d\mu_2 \right] \\ &\times \exp(-2\pi i x \nu) d\nu. \end{split}$$
(13)

It may be rewritten as a convolution in the Fourier domain:

$$\int \int \left[ \int M(\nu_1 - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right] \\ \times \exp(\pi i \lambda (z_2 - z_1) \nu_1^2) \operatorname{rect} \left( \frac{\nu_1}{\Delta \mu_2 / \lambda F} \right) \\ \times M^*(-\nu + \nu_1) \exp(2\pi i x \nu) d\nu_1 d\nu.$$
(14)

Now, we need to do additional free-space propagation of  $-z_2$ , which means another Fourier multiplying by the chirp factor and inverse Fourier:

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$$\int \int \left[ \int M(\nu_1 - \mu_1) G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) d\mu_1 \right]$$

$$\times \exp(\pi i \lambda (z_2 - z_1) \nu_1^2) \operatorname{rect} \left( \frac{\nu_1}{\Delta \mu_2 / \lambda F} \right) \qquad (15)$$

$$\times M^*(-\nu + \nu_1) \exp(-\pi i \lambda z_2 \nu^2)$$

$$\times \exp(2\pi i x \nu) d\nu_1 d\nu.$$

This is the field distribution in the output plane. Note that the masks of encoding and decoding are random and, therefore, are uncorrelated:

$$\int M(\nu)M^*(\nu-\nu_1)\mathrm{d}\nu = \delta(\nu_1). \tag{16}$$

This uncorrelation relation is very strong (the mask is very random), and it may be rewritten as

$$\int f(\nu)M(\nu)M^*(\nu-\nu_1)\mathrm{d}\nu = \delta(\nu_1) \tag{17}$$

for any general function  $f(\nu)$ . Because we are talking about fields, M can be complex and nonhermitic. We rewrite Eq. (15) as

$$\int \int G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) \exp(-\pi i \lambda z_2 \nu^2) \exp(2\pi i x \nu)$$
$$\times \left[ \int \exp(\pi i \lambda (z_2 - z_1) \nu_1^2) \operatorname{rect}\left(\frac{\nu_1}{\Delta \mu_2 / \lambda F}\right) \right]$$
$$\times M(\nu_1 - \mu_1) M^*(-\nu + \nu_1) d\nu_1 d\mu_1 d\nu, \qquad (18)$$

and use the assumption of Eq. (17), yielding

$$\int \int G(\mu_1) \exp(\pi i \lambda z_1 \mu_1^2) \exp(-\pi i \lambda z_2 \nu^2)$$
$$\times \exp(2\pi i x \nu) \delta(\nu - \mu_1) d\mu_1 d\nu, \tag{19}$$

and resulting with

$$\int G(\mu_1) \exp(\pi i \lambda (z_1 - z_2) \mu_1^2) \exp(2\pi i x \mu_1) d\mu_1.$$
 (20)

For intensity, in the spatially coherent case we may write

$$I(x) = \left| \int G(\mu_1) \exp(\pi i \lambda (z_1 - z_2) \mu_1^2) \exp(2\pi i x \mu_1) d\mu_1 \right|^2.$$
(21)

For  $z_1 = z_2$ , one obtains superresolution because the field of the output is equal to the high resolution object's field g(x). An interesting application for the proposed setup can be filtering. By choosing  $z_1 - z_2$  to not be zero, we actually apply filtering over the input object.

Note that, for the assumption of Eq. (17), the Fourier of the encoding/decoding mask M must contain a lot of features, which means that m(x) should be large in the spatial domain-at least as large as g(x) and definitely much larger than the point spread function of the imaging system before superresolution (within the width of the aperture, which was a rect in our case, the function M should have as many spatial features as possible). In addition, the spectral width of the coding/decoding mask, i.e., the width of M should be as large as the synthetic aperture we aim to generate in the superresolution process. This, in a way, resembles CDMA coding, where orthogonality is also required to separate mixed bits. In our case, the resolution of m(x), i.e., its smallest feature should be at least as small as the smallest feature in g(x) that we want to see divided by the superresolution factor. This is the additional cost for the superresolution improvement in addition to the energy and contrast losses.

#### 3. Numerical Simulation of the System

We presented three numerical simulations: one for 1D superresolution and two numerical simulations for 2D resolving. For the system simulations, we have assumed spatially coherent illumination at a wavelength of 500 nm. The size of the pixels in our input mask was 0.1 mm, and the density of the random holes in the encoding/decoding mask was 25%. An example of an encoding/decoding mask that we used in our simulations is presented in Fig. 2.

In the 1D simulation, we chose the distances  $z_1 = z_2 = 8$  m. The size of the low pass filter was 1.99 lines/mm. The width of the lines of the input object was 0.2 mm.

In the 2D simulation, we chose the distances  $z_1 = z_2 = 10$  m for the grating input and 12 m for the lattice input object. The size of the low pass filter was of 1.99 lines/mm in both axes. The width of the lines of



Fig. 2. Mask that was used for the encoding and the decoding in the numerical simulation.



Fig. 3. Numerical results for 1D superresolution: (a) high-resolution reference image; (b) image after reducing high spatial frequencies; (c) recovered image; (d) recovered image after reducing noise.

the input grating was 0.28 mm, and the size of the lattice unit was 0.2 mm  $\times$  0.2 mm.

In Fig. 3, we present the numerical simulations of the setup. In Fig. 3(a), we show the image of the high-resolution reference. In Fig. 3(b), we see the low-resolution reference as it is seen after the spatial

blurring due to a low-resolution imaging system. After applying the proposed approach, by adding the random masks, the obtained result are shown in Fig. 3(c). In Fig. 3(d), one may see the results of Fig. 3(c) after reducing the additive noises due to the processing procedure. One may see that the reconstructed image is very similar to the original high-resolution reference. The signal-to-noise ratio (SNR) of the image was improved from 3 dB in the low-resolution image of Fig. 3(b) to about 6 dB in Fig. 3(c) after adding the random mask (the superresolved reconstruction) and to 10 dB in Fig. 3(d) after reducing the additive noise.

In Fig. 4, we present two additional numerical simulations of the proposed technique for the 2D superresolution case. In Figs. 4(a)-I and 4(a)-II, we show two high-resolution input reference images. In Figs. 4(b)-I and 4(b)-II, we see the low-resolution references as they are seen after the spatial blurring due to a low-resolution imaging system. After applying the proposed approach by adding the random masks, the obtained results are seen in Figs. 4(c)-I and 4(c)-II. In Figs. 4(d)-I and 4(d)-II, we present the obtained results of Fig. 4(c)-I and 4(c)-II after reducing the additive noise generated in the processing. One may see that the reconstructed images in the 2D case are very similar to the original high-resolution references, exactly as was obtained in the 1D case. Also in the 2D images, the SNR has been improved. In the case of Fig. 4(b)-I, the SNR improvement is from 2.9 to 3.4 dB after the addition of the random mask yielding the superresolved image of Fig. 4(c)-I and to 6.7 dB after reducing the additive noise as seen in Fig. 4(d)-I. For the second image, the improvement in the SNR was from 2.9 dB in Fig. 4(b)-II to 3.5 dB in Fig. 4(c)-II after adding the random mask (superresolved image) and to 10.22 dB after reducing the additive noise as obtained in Fig. 4(d)-II.



Fig. 4. Numerical results for 2D superresolution [(I) 2D input grating and (II) 2D input lattice]: (a) high-resolution reference image; (b) image after reducing high spatial frequencies; (c) recovered image; (d) recovered image after reducing noise.

# 4. Experimental Results

To validate the proposed approach working under incoherent illumination, the optical setup shown in Fig. 5 was assembled at the laboratory. Extended (nonpunctual) polychromatic (white light) illumination is provided by Fiber-Lite MI-150 fiber optic illuminator (halogen lamp source focused onto a fiber optic light guide). For the encoding/decoding process, two identical binary amplitude square random masks with different magnifications are used in the experiment. Figure 6 depicts the area of the masks that is used for encoding/decoding the object's angular information and where the black circle acts as a reference detail. The masks are fabricated by photolithography on chrome on glass substrate. The encoding mask (M1) has a pixel size of 3  $\mu$ m and a total width of 4.5 mm, while the decoding one (M2) has a pixel size of 20  $\mu$ m and a total with of 30 mm. Thus, the corresponding mask magnification is set to be 6.67.

Two imaging modules compose the experimental setup. In the first one, a variable circular diaphragm is attached to the back focal plane of a commercial microscope lens having 0.1 NA. The diaphragm allows us to stop down the resolution of the objective lens to match its NA with the size of M1 used in the experiment. The magnification of the first imaging system must be properly adjusted to be equal to that one defined by both random masks. Otherwise, no superresolution effect will be attainable. To allow this, the first imaging system is placed onto a micrometer stage to allow magnification adjustment between the M1 and M2 planes.

Figure 7 depicts the cases without and with proper magnification matching between the masks. The white circle is for referencing both images and also for Fig. 6. Since the input object is placed before M1, its image will be placed also in a plane previous to plane M2. Thus, the second imaging module images the aerial image provided by the first system through M2. A photographic objective with variable focus (or magnification) is selected as the second imaging module to magnify the aerial image into the output



Fig. 5. (Color online) Experimental setup for incoherent illumination case.



Fig. 6. Picture of the theoretical design of the random mask used in the experiment (only a small part is included). The black circle marks a mask's detail that can also be traced in Fig. 7 for reference.

plane where the CCD (Basler A312f,  $582 \times 782$  pixels,  $8.3 \ \mu m$  pixel size, 12 bits/pixel) is placed. This second imaging module plays the role of the tube lens used in microscope systems. Because of the magnification ratio between the two imaging modules of the setup, M2 could be a low-frequency mask (higher pixel size than M1, as it was previously described)



Fig. 7. Example of (a) magnification mismatch and (b) perfect magnification adjustment between the two masks. White circle marks the same area for reference.



Fig. 8. Experimental results: (a) without and (b) with using the proposed approach and corresponding with conventional low-resolution and superresolved images, respectively.

and there is no need for high-resolution optics in the second imaging module.

Under these assumptions, we perform our superresolution approach. A positive USAF resolution test target is used as the input object. The circular diaphragm of the first imaging lens is closed to stop down the resolution of the experimental setup. Figure 8(a) depicts the low-resolution image provided by the experimental setup where Group 6—Element 3 (G6-E3 from now on) is the last-resolved element in the test that defines a resolution limit that is equal to 12.4  $\mu$ m (80.6 lp/mm). This resolution limit corresponds with a theoretical value of 0.022 NA in the first imaging module, considering the central wavelength (0.55  $\mu$ m) of the illumination. After performing the superresolved approach, the resolution is improved until G7-E2 corresponding with 6.9  $\mu$ m (144.0 lp/mm), as one can see in Fig. 8(b), which defines a resolution gain factor equal to 1.8.

Because the pixel size in *M*1 has a width of  $3 \mu m$ , the expected theoretical resolution limit is twice the pixel width, that is, 6  $\mu$ m (or 166.7 lp/mm). This resolution limit corresponds in the USAF test with G7-E3 (161 lp/mm), which is very close to the theoretical limit, and it is not resolved due to experimental factors, such as noise, contrast reduction, mismatch between masks, etc. However, in any case, this is the best resolution limit that can be achieved using the proposed approach: one defined by the minimum period of the encoding mask. Such a minimum resolution limit is theoretically independent from the NA of the first imaging module. Then, our purpose is to demonstrate this theoretical assumption. Figure 9 depicts the experimental results achieved with different diameters of the limiting diaphragm. Running from left to right, the NA value is increased from 0.016 to 0.022 and to 0.031, and the resolution limit is improved from 17.5  $\mu$ m [G5-E6 in Fig. 9(a)], 12.4  $\mu$ m [G6-E3 in Fig. 9(b)], and 8.8  $\mu$ m [G6-E6 in Fig. 9(c)] to 6.9  $\mu$ m [G7-E2 in Fig. 9(d)] and 6.2  $\mu$ m [G7-E3 in Figs. 9(e) and 9(f)]. The corresponding resolution gain factors are 2.5, 2, and 1.4, respectively. Thus, it is demonstrated that the resolution limit of



Fig. 9. Experimental results showing that the gain in resolution does not depend on the NA of the first module imaging system: (a)(d), (b) (e), and (c)(f) depict different cases of low and superresolved images corresponding with different diameters of the lens diaphragm.

the setup is defined by the minimum pixel size of the encoding mask M1.

## 5. Conclusions

In this paper, we have presented a new direction for superresolution imaging involving two static amplitude random masks that do not have to be in contact with the imaged object. The outcome is a superresolved image in which one does not have restrictions either in time or in field of view but rather only in the dynamic range of the detection imaging device. We have numerically simulated and experimentally validated the proposed concept for coherent and incoherent illuminations, respectively. In the simulations and the experimental validation, we considered a 4F imaging processor (unity magnification) and a low NA imaging setup (adjustable arbitrary magnification), respectively.

One of the main advantages of the reported approach is that the achieved spatial-resolution limit does not depend on the NA of the imaging lens but rather it is a function of the minimum pixel size of the encoding mask. Because the encoding mask is random, its period varies continuously, starting from the double of the minimum pixel size of the encoding mask. Thus, on one hand, there is no need to match the mask period to the NA of the imaging lens, as happens in other similar approaches using diffraction gratings. On the other hand, the resolution gain is not limited to a given value because the final resolution limit depends on the encoding mask pixel size and a limiting factor incoming from noise.

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#### References

- E. Abbe, "Beitrage zur theorie des mikroskops und der mikroskopischen wahrnehmung," Arch. Mikrosk. Anat. 9, 413–468 (1873).
- Z. Zalevsky, D. Mendlovic, and A. W. Lohmann, "Optical system with improved resolving power," in *Progress in Optics*, E. Wolf, ed. (Elsevier, 1999), Vol. XL, Chap. 4.
- 3. Z. Zalevsky and D. Mendlovic, *Optical Super Resolution* (Springer, 2002).
- M. Françon, "Amélioration the résolution d'optique," Il Nuovo Cimento Suppl. 9, 283–290 (1952).
- 5. W. Lukosz, "Optical systems with resolving powers exceeding the classical limits," J. Opt. Soc. Am. 56, 1463–1472 (1966).
- A. Shemer, D. Mendlovic, Z. Zalevsky, J. García, and P. García-Martínez, "Superresolving optical system with time multiplexing and computer decoding," Appl. Opt. 38, 7245–7251 (1999).
- M. A. Grimm and A. W. Lohmann, "Super resolution image for one-dimensional objects," J. Opt. Soc. Am. 56, 1151–1156 (1966).
- 8. W. Lukosz, "Optical systems with resolving powers exceeding the classical limits. II," J. Opt. Soc. Am. 57, 932–941 (1967).

- A. Bachl and W. Lukosz, "Experiments on superresolution imaging of a reduced object field," J. Opt. Soc. Am. 57, 163-164 (1967).
- Z. Zalevsky, E. Leith, and K. Mills, "Optical implementation of code division multiplexing for super resolution. Part I. Spectroscopic method," Opt. Commun. 195, 93–100 (2001).
- Z. Zalevsky, E. Leith, and K. Mills, "Optical implementation of code division multiplexing for super resolution. Part II. Temporal method," Opt. Commun. 195, 101–106 (2001).
- J. Solomon, Z. Zalevsky, and D. Mendlovic, "Super resolution using code division multiplexing," Appl. Opt. 42, 1451–1462 (2003).
- W. Gartner and A. W. Lohmann, "An experiment going beyond Abbe's limit of diffraction," Z. Phys. 174, 18–24 (1963).
- 14. A. W. Lohmann and D. P. Paris, "Superresolution for nonbirefringent objects," Appl. Opt. 3, 1037–1043 (1964).
- A. Zlotnik, Z. Zalevsky, and E. Marom, "Superresolution with nonorthogonal polarization coding," Appl. Opt. 44, 3705–3715 (2005).
- I. Kartashev, "Optical systems with enhanced resolving power," Opt. Spectrosc. 9, 204–206 (1960).
- D. Mendlovic, J. Garcia, Z. Zalevsky, E. Marom, D. Mas, C. Ferreira, and A. W. Lohmann, "Wavelength multiplexing system for a single mode image transmission," Appl. Opt. 36, 8474–8480 (1997).
- J. García, V. Micó, D. Cojoc, and Z. Zalevsky, "Full field of view super-resolution imaging based on two static gratings and white light illumination," Appl. Opt. 47, 3080–3087 (2008).
- Z. Zalevsky, P. García-Martínez, and J. García, "Superresolution using gray level coding," Opt. Express 14, 5178–5182 (2006).
- D. Mendlovic, A. W. Lohmann, N. Konforti, I. Kiryuschev, and Z. Zalevsky, "One dimensional superresolution optical system for temporally restricted objects," Appl. Opt. 36, 2353–2359 (1997).
- D. Mendlovic, I. Kiryuschev, Z. Zalevsky, A. W. Lohmann, and D. Farkas, "Two dimensional super resolution optical system for temporally restricted objects," Appl. Opt. 36, 6687–6691 (1997).
- D. Mendlovic, D. Farkas, Z. Zalevsky, and A. W. Lohmann, "High-frequency enhancement by an optical system for superresolution of temporally restricted objects," Opt. Lett. 23, 801–803 (1998).
- A. Shemer, Z. Zalevsky, D. Mendlovic, N. Konforti, and E. Marom, "Time multiplexing super resolution based on interference grating projection," Appl. Opt. 41, 7397–7404 (2002).
- Z. Zalevsky, J. García, and V. Micó, "Transversal superresolution with noncontact axial movement of periodic structures," J. Opt. Soc. Am. A 24, 3220–3225 (2007).
- V. Mico, O. Limon, Z. Zalevsky, and J. García, "Transversal resolution improvement using rotating-grating timemultiplexing approach," J. Opt. Soc. Am. A 25, 1115–1129 (2008).
- H. Bartelt and A. W. Lohmann, "Optical processing of onedimensional signals," Opt. Commun. 42, 87–91 (1982).
- Z. Zalevsky, D. Mendlovic, and A. W. Lohmann, "Super resolution optical systems for objects with finite sizes," Opt. Commun. 163, 79–85 (1999).
- E. Sabo, Z. Zalevsky, D. Mendlovic, N. Komforti, and I. Kiryushev, "Superresolution optical system with two fixed generalized Damman gratings," Appl. Opt. **39**, 5318–5325 (2000).
- E. Sabo, Z. Zalevsky, D. Mendlovic, N. Komforti, and I. Kiryushev, "Superresolution optical system using three fixed generalized gratings: experimental results," J. Opt. Soc. Am. A 18, 514–520 (2001).

- Z. Zalevsky, V. Eckhouse, N. Konforti, A. Shemer, D. Mendlovic, and J. García, "Super resolving optical system based on spectral dilation," Opt. Commun. 241, 43–50 (2004).
- J. Garcia, Z. Zalevsky, and D. Fixler, "Synthetic aperture superresolution by speckle pattern projection," Opt. Express 13, 6073–6078 (2005).
- 32. E. Ben-Eliezer and E. Marom, "Aberration-free superresolution imaging via binary speckle pattern encoding

and processing," J. Opt. Soc. Am. A 24, 1003–1010 (2007).

- Z. Zalevsky, E. Saat, S. Orbach, V. Mico, and J. Garcia, "Exceeding the resolving imaging power using environmental conditions," Appl. Opt. 47, A1–A6 (2008).
- 34. Z. Zalevsky, E. Fish, N. Shachar, Y. Vexberg, V. Micó, and J. Garcia, "Super-resolved imaging with randomly distributed, time- and size-varied particles," J. Opt. A Pure Appl. Opt. 11, 085406 (2009).